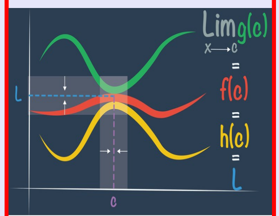
Math 261

Spring 2023

Lecture 19



Feb 19-8:47 AM

Class QZ 4:

1) Evaluate $\lim_{x \to \infty} \frac{5 - 2x}{4x + 1} = \frac{-\infty}{\infty}$ I.F.

$$\lim_{x \to \infty} \frac{5 - 2x}{4x + 1} = \lim_{x \to \infty} \frac{\frac{5}{x} - \frac{2x}{x}}{\frac{4x}{x} + \frac{1}{x}} = \lim_{x \to \infty} \frac{\cancel{\frac{5}{x}}^{\to 0} - 2}{4 + \cancel{\frac{1}{x}}^{\to 0}} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$$

2) Find $f'(x)$ for $f(x) = \frac{2x^2}{x^2 - 1}$

$$f'(x) = \frac{4x(x^2-1) - 2x^2(2x)}{(x^2-1)^2} = \frac{4x^3 - 4x - 4x^3}{(x^2-1)^2}$$

$$= \boxed{\frac{-4x}{(x^2-1)^2}}$$

Mar 8-9:35 AM

Recall

1) $\lim_{h \to 0} \frac{\sin h}{h} = 1$

2) $\lim_{h \to 0} \frac{1 - \cos h}{h} = 0$

3) $\frac{d}{dx}[\sin x] = \cos x$

4) $\frac{d}{dx}[\cos x] = -\sin x$

5) $\frac{d}{dx}[\tan x] = \sec^2 x$

6) $\frac{d}{dx}[\cot x] = -\csc^2 x$

7) $\frac{d}{dx}[\sec x] = \sec x \tan x$

8) $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Mar 9-8:51 AM

$y = x^2 \sin x$

$$\boxed{y' = 2x \cdot \sin x + x^2 \cdot \cos x}$$

$y'' = 2[1 \cdot \sin x + x \cdot \cos x] + 2x \cdot \cos x + x^2 \cdot -\sin x$

$y'' = 2\sin x + 2x \cos x + 2x \cos x - x^2 \sin x$

$$\boxed{y'' = 2 \sin x + 4x \cos x - x^2 \sin x}$$

Mar 9-8:55 AM

$$f(x) = \frac{\sec x}{1 + \tan x}$$

$$f'(x) = \frac{\frac{d}{dx}[\sec x]\cdot(1+\tan x) - \sec x \cdot \frac{d}{dx}[1+\tan x]}{(1+\tan x)^2}$$

$$= \frac{\sec x \tan x\,(1+\tan x) - \sec x \cdot \sec^2 x}{(1+\tan x)^2}$$

$$= \frac{\sec x\,[\tan x + \tan^2 x - \sec^2 x]}{(1+\tan x)^2}$$

Recall

$$1 + \tan^2 x = \sec^2 x$$

$$= \frac{\sec x\,[\tan x + \cancel{\tan^2 x} - 1 - \cancel{\tan^2 x}]}{(1+\tan x)^2}$$

$$f'(x) = \frac{\sec x\,[\tan x - 1]}{(1+\tan x)^2}$$

Mar 9-8:59 AM

$$y = x^3 \sin x \tan x$$

$$y' = \frac{d}{dx}\left[x^3 \sin x \tan x\right]$$

$$= \frac{d}{dx}[x^3] \cdot \sin x \tan x + x^3 \cdot \frac{d}{dx}[\sin x \tan x]$$

$$= 3x^2 \cdot \sin x \tan x + x^3 \cdot [\cos x \cdot \tan x + \sin x \cdot \sec^2 x]$$

$$= 3x^2 \sin x \tan x + x^3 \cdot \underbrace{\cos x \cdot \tan x}_{\sin x} + x^3 \sin x \sec^2 x$$

$$y' = 3x^2 \sin x \tan x + x^3 \sin x + x^3 \sin x \sec^2 x$$

$$\boxed{y' = x^2 \sin x\,[3 \tan x + x + x \sec^2 x]}$$

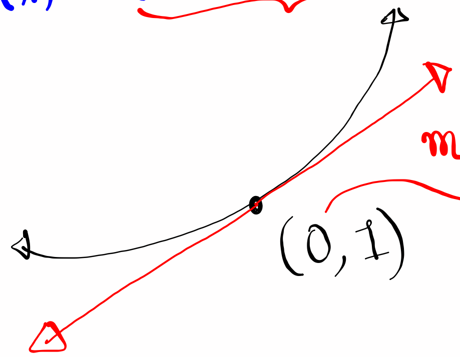
Mar 9-9:05 AM

Find eqn of tan. line to the graph of

$f(x) = \underbrace{(x+1)\cdot \cos x}$ at $x=0$.

$f(0) = (0+1)\cdot \cos 0$

$= 1 \cdot 1 = 1$



$m = f'(0)$

$f'(x) = 1\cdot \cos x + (x+1)\cdot -\sin x$

$f'(0) = 1 \cdot \cos 0 + \cancel{(0+1)\cdot -\sin 0}$

$m = f'(0) = 1$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 0) \rightarrow \boxed{y = x + 1}$

Mar 9-9:11 AM

Find all points $\rightarrow (x,y)$ where the curve $y = \dfrac{\cos x}{2+\sin x}$ has $\underbrace{\text{horizontal tan. line.}}_{m=0}$

Solve $y' = 0$ $\quad y' = \dfrac{\frac{d}{dx}[\cos x]\cdot(2+\sin x) - \cos x \cdot \frac{d}{dx}[2+\sin x]}{(2+\sin x)^2}$

$$y' = \frac{-\sin x(2+\sin x) - \cos x \cdot \cos x}{(2+\sin x)^2} = \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$y' = \frac{-2\sin x - \overbrace{(\sin^2 x + \cos^2 x)}^{1}}{(2+\sin x)^2} = \frac{-2\sin x - 1}{(2+\sin x)^2}$$

$y' = 0 \rightarrow -2\sin x - 1 = 0 \rightarrow \sin x = \frac{-1}{2}$

Ref. Angle $= 30°$ $\frac{\pi}{6}$

QIII $\Rightarrow x = \text{R.A.} + \pi + 2n\pi$

$= \frac{\pi}{6} + \pi + 2n\pi = \boxed{\frac{7\pi}{6} + 2n\pi}$

QIV $\Rightarrow x = 2\pi - \text{RA.} + 2n\pi$

$= 2\pi - \frac{\pi}{6} + 2n\pi = \boxed{\frac{11\pi}{6} + 2n\pi}$

$\left(\frac{7\pi}{6} + 2n\pi, f\left(\frac{7\pi}{6} + 2n\pi\right)\right) = \left(\frac{7\pi}{6} + 2n\pi, \frac{-\sqrt{3}}{3}\right)$

$$y = \frac{\cos\frac{7\pi}{6}}{2+\sin\frac{7\pi}{6}} = \frac{\cos 210°}{2+\sin 210°} = \frac{-\cos 30°}{2-\sin 30°} = \frac{-\frac{\sqrt{3}}{2}}{2-\frac{1}{2}}$$

$\left(\frac{11\pi}{6} + 2n\pi, f\left(\frac{11\pi}{6} + 2n\pi\right)\right) = \left(\frac{11\pi}{6} + 2n\pi, \frac{\sqrt{3}}{3}\right)$

$$y = \frac{\cos\frac{11\pi}{6}}{2+\sin\frac{11\pi}{6}} = \frac{\cos 30°}{2-\sin 30°} = \frac{\frac{\sqrt{3}}{2}}{2-\frac{1}{2}} =$$

Mar 9-9:15 AM

Evaluate $\lim_{x\to 0} \dfrac{\sin 7x \sin 9x}{63x^2} = \dfrac{0}{0}$ I.F.

$$\lim_{x\to 0} \frac{\sin 7x \sin 9x}{63x^2} = \lim_{x\to 0} \frac{\sin 7x \cdot \sin 9x}{7x \cdot 9x}$$

$$= \underbrace{\boxed{\lim_{x\to 0} \frac{\sin 7x}{7x}}}_{\to 1} \cdot \underbrace{\boxed{\lim_{x\to 0} \frac{\sin 9x}{9x}}}_{\to 1}$$

$$= 1 \cdot 1 = \boxed{1}$$

Mar 9-9:32 AM

Find $\lim_{x\to \frac{\pi}{4}} \dfrac{1-\tan x}{\sin x - \cos x} = \dfrac{1-\tan\frac{\pi}{4}}{\sin\frac{\pi}{4} - \cos\frac{\pi}{4}}$

$x \to 45°$

$$= \frac{1-1}{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}} = \frac{0}{0} \text{ I.F.}$$

$$\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x - \cos x} = \lim_{x\to \frac{\pi}{4}} \frac{1 - \frac{\sin x}{\cos x}}{\sin x - \cos x} = \lim_{x\to \frac{\pi}{4}} \frac{\cancel{\cos x - \sin x}}{\cos x(\cancel{\sin x - \cos x})}$$

LCD = $\cos x$

$$= \lim_{x\to \pi/4} \frac{-1}{\cos x} = \frac{-1}{\cos \pi/4} = \frac{-1}{\frac{\sqrt{2}}{2}} = \frac{-\cancel{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{-\sqrt{2}}$$

≈ -1.414

Evaluate $\dfrac{1-\tan x}{\sin x - \cos x}$

at $x = 45.1°$ $\quad \dfrac{1-\tan 45.1°}{\sin 45.1° - \cos 45.1°} \approx -1.417$

at $x = 45.01°$ $\quad \dfrac{1-\tan 45.01°}{\sin 45.01° - \cos 45.01°} \approx -1.414$

Look into Chain Rule

Mar 9-9:39 AM